

# Derivation of Displacement-Based Fragility Functions for Masonry Buildings

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## ABSTRACT:

A nonlinear static procedure is presented for the derivation of displacement-based fragility functions for seismic performance assessment of masonry buildings on regional scale. The procedure makes use of the existing displacement-based method calibrated for masonry buildings in the present research work. For a specified performance level, the method compares the displacement capacity of the structural systems with the displacement demand at the secant vibration period of the systems taking into account the energy dissipation characteristics with the expected variability in geometrical and mechanical properties of the systems in order to assess their seismic vulnerability. The displacement demand on structures is defined using random 5% damped linear displacement response spectrum. The probability of exceeding different specified performance limit states of buildings is assessed and presented in terms of displacement-based fragility functions, which gives the probability of exceedance of different specified damage states for a given median displacement demand which is obtained from each random displacement spectrum using the median capacity parameters of the buildings. The derived fragility functions also take into account the record-to-record variability explicitly in the building capacity parameters obtained from the nonlinear dynamic analyses. The derived fragility functions can be used for the intensity-based, scenario-based and time-based (annualized losses) assessment on regional scale. The method is applied to the urban masonry building stock of Pakistan.

*Keywords: Displacement-Based; Fragility Functions; Performance Assessment; Masonry Buildings.*

## 1. INTRODUCTION

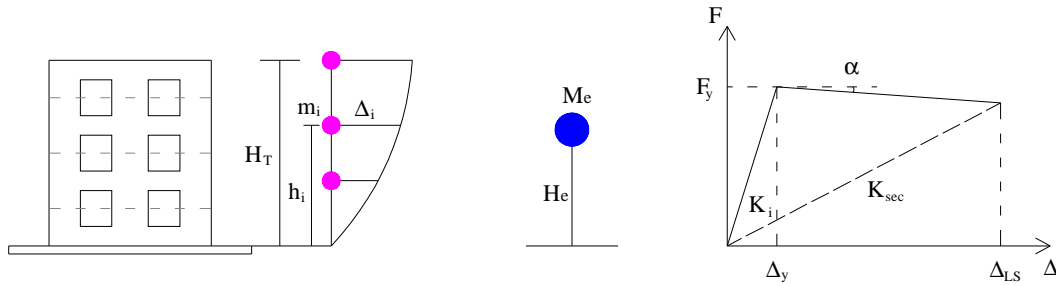
The paper presents an analytical nonlinear static displacement-based method, capable of incorporating sources of expected uncertainties in the seismic demand and structural capacity explicitly, for the derivation of fragility functions of groups of structures. The method makes use of the basic principles of the mechanics of materials and structures to assess the seismic performance of structural systems with relatively high computational ease. The mechanical nature of the method makes it suitable for the assessment of structures within the context of regional earthquake loss estimation, real-time loss modelling, design of insurance schemes, code calibration and efficient design of retrofit schemes.

Conceptually the method can be employed to any type of structural system, the present study applied it to the existing unreinforced fired solid brick masonry buildings with reinforced concrete slab, which is the most prevailing typology of existing building stock within Europe and other developing parts of the world e.g. Pakistan [Ali, 2006]. Application to code-designed buildings is also performed. Fragility functions are derived for the case study building classes considering the global response of the buildings, which can be used later in the seismic performance assessment of these buildings on a regional scale for earthquake loss estimation, for retrofit schemes and new design procedures to meet a given target performance level.

## 2. DISPLACEMENT-BASED FRAGILITY FUNCTIONS

### 2.1. Nonlinear Static SDOF Systems for Masonry Buildings

The seismic response of masonry buildings with reinforced concrete slab, with or without ring/bond beams, is mainly governed by the global mechanism and shear response of the in-plane walls with limited energy dissipation capabilities and ductility level. An equivalent single degree of freedom (SDOF) system is used to simulate the nonlinear response of an actual building in terms of displacement capacity at different performance levels, limit states, see Fig. 2.2. for an SDOF idealization of masonry buildings.



**Figure 2.2.** Nonlinear static SDOF idealization of masonry buildings

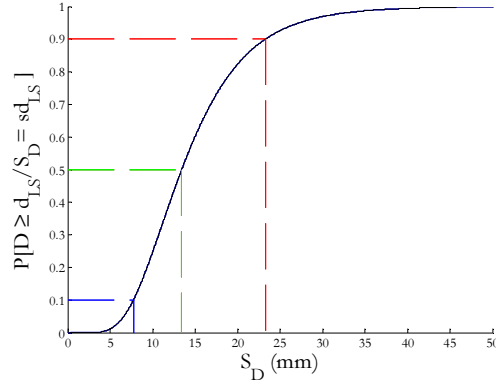
In this figure,  $H_T$  represents the total building height;  $h_i$  represents the  $i^{\text{th}}$  floor height,  $\Delta_i$  represents the lateral displacement and  $m_i$  represents the  $i^{\text{th}}$  floor mass for a given deformed shape of the building;  $M_e$  and  $H_e$  represent the mass and height of the equivalent SDOF system;  $\Delta_y$  and  $\Delta_{LS}$  represent the equivalent yield and ultimate limit state displacement that represents the displacement capacity of the actual building at the center of seismic force for a specified deformed shape;  $K_i$  represents the initial pre-yield stiffness;  $F_y$  represents the yielding force;  $K_{sec}$  represents the secant stiffness; and  $\alpha$  represents the ratio of post- to pre-yield stiffness. For seismic assessment, the static SDOF system is completely defined by secant vibration period, limit state displacement capacity and energy dissipation characteristics of buildings represented as viscous damping.

### 2.2. Methodology for the Derivation of Fragility Functions

The method makes use of the SDOF systems that are representative of the building stock in terms of their characteristic mechanical properties: secant vibration period, displacement capacity and energy dissipation capability, considering all possible uncertainties in geometrical and material properties of the structural systems as well as the uncertainties introduced by real earthquake loading in the capacity parameters. Controlled Monte Carlo simulation is used to generate thousands of random populations of the above parameters, representing regional building stock, defined a priori using a complete probabilistic distribution with prescribed mean and c.o.v. The seismic demand on buildings is defined in terms of randomly generated 5% damped linear displacement response spectra. The vulnerability of the generated building stock is assessed using displacement-based fully probabilistic approach developed for R.C. buildings by Crowley *et al.* [2004] and calibrated for masonry by Ahmad *et al.* [2010].

For each of the generated random spectrum, the displacement capacity of the SDOF systems for a specified limit state, buildings performance level, are compared with the displacement demand at the secant vibration periods of the systems. The number of buildings having capacity less than the demand over the total population of the buildings gives an estimate of the limit state exceedance probability of the systems. Spectral reduction is performed for higher limit states in order to take into account the structural nonlinear effects. The considered spectrum is used along with the median capacity parameters, mechanical parameters with 50 percent probability of exceedance on a regional scale, mainly yield vibration period, yield displacement capacity and viscous damping to assess the median

displacement demand i.e. performance point (pp). For each limit state, and each spectrum, the probability of exceedance is plotted against the median displacement demand, see Fig. 2.1 for an exemplificative fragility function for near-collapse limit state of Pakistani two storey unreinforced masonry building class with solid burnt clay brick work in cement mortar with reinforced concrete slab.



**Figure 2.1.** Example displacement-based fragility function for near-collapse limit state of two storey buildings.

The fragility function shows that for this class of buildings 10 percent of the total building stock will be collapsed at the attainment of displacement demand of 7.75 mm, 50 percent buildings at the attainment of displacement demand of 13.50 mm and 90 percent buildings at the attainment of displacement demand of 23.30 mm. Fragility functions developed in similar fashion for other classes of buildings can be used efficiently to derive damage probability matrices (DPM) for seismic performance assessment of buildings on regional scale with fairly high computational ease.

### 3. CASE STUDY BUILDINGS

#### 3.1. Existing Urban Unreinforced Masonry Building Stock of NE Pakistan

In the urban areas of NE Pakistan the most popular residential building construction is masonry with solid burnt clay brick units and cement mortar with an additional ingredient called khaka [Ali, 2006]. Mostly the buildings are two to three storeys with less number of buildings with higher storeys, maximum five storeys. The buildings are provided with lintel and ring beams with reinforced concrete slab. The predominant mechanism of these buildings is global in-plane shear mechanism with diagonal shear cracks accompanied by horizontal bed joint sliding cracks, as observed in the experimental tests [Ali and Naeem, 2007] and real earthquake excitation [Javed *et al.*, 2006]. Ahmad *et al.* [2010] performed dynamic analysis of the 3D buildings, representative of residential building stock of NE Pakistan, with actual material properties obtained from laboratory tests to derive model for the secant vibration period of these buildings:

$$T_{LS} = T_y \cdot \left( \frac{\mu}{1 + \alpha\mu - \alpha} \right)^{\frac{1}{2}} \quad (3.1)$$

$$T_y = 0.05 \cdot H^{0.75} \quad (3.2)$$

where  $T_{LS}$  represents the secant vibration period at a given post-yield limit state;  $T_y$  represents the yield vibration period;  $\mu$  represents the ductility level;  $\alpha$  represents the post-yield stiffness and strength degrading rate;  $H$  represents the total height of the building. Additionally, Ahmad *et al.* [2010] analysed the experimental data on the cyclic response of masonry shear walls and developed models for the displacement capacity and viscous damping of masonry buildings:

$$\Delta_y = \theta_y \cdot k_1 \cdot H \quad (3.3)$$

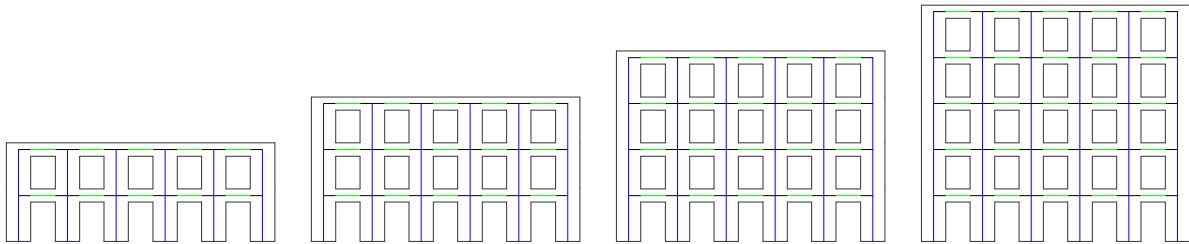
$$\Delta_{LS} = \theta_y \cdot k_1 \cdot H + (\theta_{LS} - \theta_y) k_2 \cdot h_s \quad (3.4)$$

$$\xi = 5 + \frac{27 \cdot (\mu - 1)}{\mu \cdot \pi} \quad (3.5)$$

where  $\Delta_y$  represents the crack/yield displacement capacity;  $\Delta_{LS}$  represents the post-yield limit state displacement capacity;  $\theta_y$  represents the crack/yield drift limit;  $\theta_{LS}$  represents the post-yield limit state drift limit;  $h_s$  represents the interstorey height of the building;  $k_1$  and  $k_2$  represents the height reduction coefficients used to obtain the displacement capacity at the center of the seismic force;  $\xi$  represents the viscous damping of the system. The derived mean drift levels are: 0.088% with a c.o.v. of 8.8% at the formation of diagonal cracks in the walls, 0.22% with a c.o.v. of 35% for the maximum response of walls i.e. at the attainment of maximum shear strength in the walls, and 0.46% with a c.o.v. of 27% at the near collapse limit state of the wall. A drift level of 0.13% can be obtained for the yield limit state of the shear walls using 50% cracked section properties with effective shear area of 70% of the gross area with the assumption that the shear strength at cracking limit state is 60% of the maximum wall strength [Tomazevic, 1999]. The yield drift reduces to 0.11% for the consideration of crack-shear strength as 70% of the maximum wall strength.

### 3.2. Masonry Buildings Designed to Different Code Levels

The study also considered buildings designed to different code levels for their performance evaluation within the context of retrofit schemes. Four 2D case study buildings, two to five storeys Fig. 3.1., are considered and designed to five PGA levels: 0.10g, 0.15g, 0.20g, 0.25g, 0.30g using building code spectra for soil site, Type I C soil of EC-8 [CEN, 2004] and ground storey mechanism. For a given building the design PGA can be increased by stiffening floors, changing the dimensions of masonry resisting walls, pre-compression level, using horizontal/vertical reinforcement, and using confining elements [Tomazevic, 1999]. These different hypotheses increase the stiffness and strength of the system and possibly the ultimate ductility level as well.

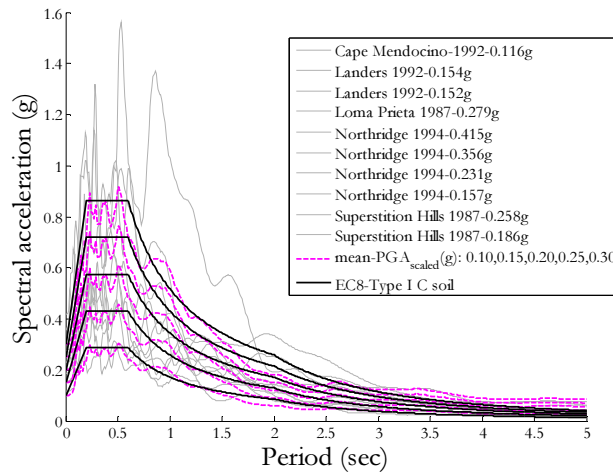


**Figure 3.1.** Case study building models designed to different code levels.

#### 3.2.1. Yield Vibration Period of Designed Buildings

The designed case study buildings are dynamically analysed for a suite of 10 real accelerograms obtained from PEER NGA data base with the mean acceleration spectrum compatible to the EC-8 elastic design spectrum for Type I C soil, see Fig. 3.1. The accelerograms are linearly scaled to exceed the yielding limit state of the masonry walls at ground floor. The aim of the dynamic analysis is to compute the vibration period of the system at the crack formation of ground floor walls with due consideration of record-to-record variability in the evaluation. The buildings are analysed using equivalent frame approach (SD-SAM) proposed by Ahmad *et al.* [2010]. The nonlinear response is considered only for the masonry walls while the response of spandrels is considered to be elastic. This modelling assumption is made due to the fact that the new designed buildings are provided with rigid slab having ring beams above the spandrels and lintel beams below the spandrels due which the

spandrels do not crack even during strong ground shaking [Ali and Naeem, 2007; Javed *et al.*, 2006]. A bi-linear hysteretic nonlinear response with Takeda type hysteresis with beta of 0.6 as proposed by Ahmad *et al.* [2010] is considered for the masonry walls.

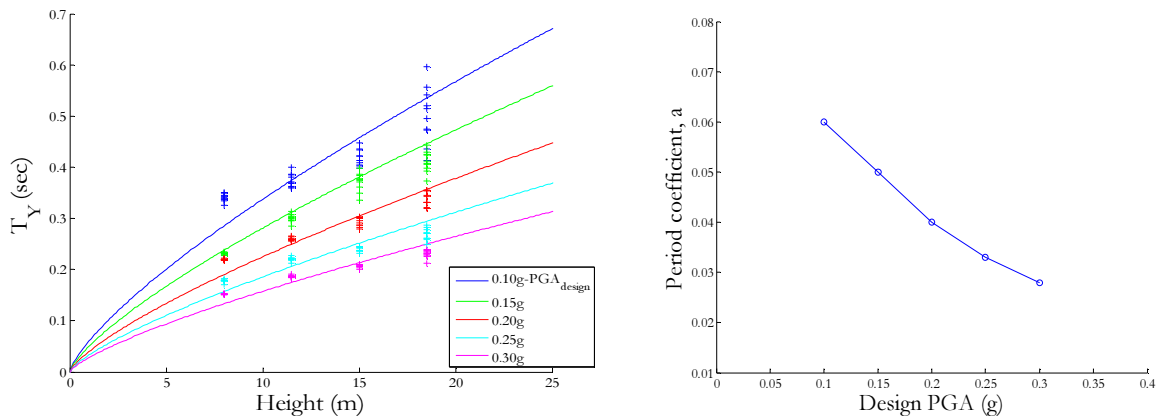


**Figure 3.1.** Mean acceleration response spectra of natural accelerograms used for dynamic analysis.

For each of the case study building, the floor displacements and the corresponding base shear force is obtained for each accelerograms and transferred to the equivalent properties, as proposed by Ahmad *et al.* [2010] in order to compute the vibration period at the cracked limit state of the ground floor. Fig. 3.2. shows the vibration periods for all the considered case study buildings and a constrained best fitting using the empirical expression of Eqn. (3.6):

$$T_y = a.H^{0.75} \tag{3.6}$$

where  $H$  represents the height of the building;  $a$  represents the period coefficient obtained from the best fitting to the data. The scatter in the period value increases with increasing height for a given design PGA and decreasing with increasing design PGA for a given building. A maximum of 12% c.o.v. of variation is observed for the mean value of  $a$ . The analyses show that how strongly the coefficient  $a$  is dependent on the design PGA level. For the present case the coefficient  $a$  values are: 0.06 for 0.10g, 0.05 for 0.15g, 0.04 for 0.20g, 0.033 for 0.25g and 0.028 for 0.30g are obtained which can be used in the performance assessment of the considered buildings. The recommendations of Crowley and Pinho [2008] for the vibration period of masonry buildings match to the building period value designed to PGA level of 0.10g.



**Figure 3.2.** Yield Vibration period of the case study building (left), period coefficient as a function of design PGA (left).

Additionally, 10 case study two storey buildings having different material properties designed to PGA level of 0.10g are considered in order to investigate the effect of material variability on the vibration period estimation. The main material properties considered are: compressive strength of masonry, modulus of elasticity and masonry diagonal tension strength. The variation in the material is considered following the empirical correlations among these parameters [Tomazevic, 1999]. The modulus of elasticity was taken as 350 times the masonry compressive strength while the diagonal tensile strength is considered as 4% of the compressive strength. Such considerations meet to the requirements of existing urban masonry building stock of Pakistan [Ahmad *et al.*, 2010]. The results of the considered case study buildings are shown in Fig. 3.3. where it can be seen clearly that the material variability does not affect significantly the mean value of period coefficient  $\alpha$ . An increase of masonry compressive strength from 2 to 7 MPA will cause the period coefficient to reduce only by about 5%.

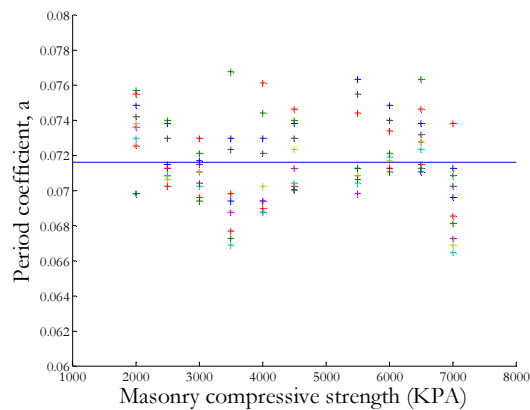


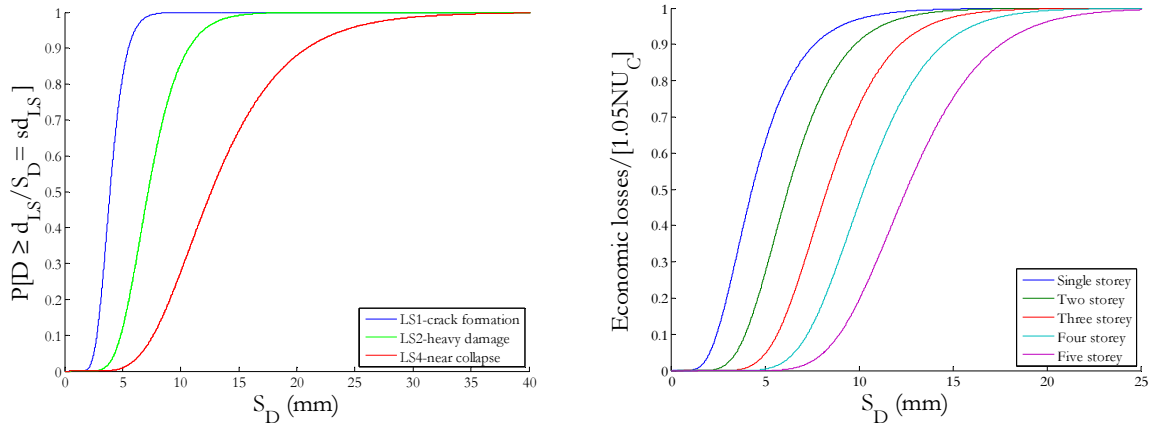
Figure 3.3. Period coefficient of two storey building with varying material properties.

## 4. FRAGILITY FUNCTIONS

### 4.1. Existing Building Stock

Controlled Monte Carlo simulation is used to generate thousands of single degree of freedom systems, representative of the existing building stock from one to five storeys, having different yield vibration period defined by Eqn. (3.1) & (3.2) with  $\alpha$  equal to -0.05 [Ahmad *et al.*, 2010]. The buildings are considered with ground floor soft-storey mechanism with three limit states of cracking, heavy damage in walls and near-collapse limit state using the experimental mean drift limits. The ground floor height is considered with a mean value of 3 m with 10% c.o.v. between 2.5 to 3.5 m. Similarly, the interstorey height is considered with a mean value of 2.5 m with 10% c.o.v. between 2.0 to 3.0 m. The equivalent height coefficients,  $k_1$  and  $k_2$ , are taken from the work of Restrepo-Velez and Magenes [2004] with 10% c.o.v. to take into account the record-to-record variability in displacement capacity. The spectral reduction factor proposed by Priestley *et al.* [2007] is used to reduce the elastic spectra for post-yield limit states. Fragility functions are derived for three damage limit states, cracking, heavy damages and near-collapse states using the procedure described earlier.

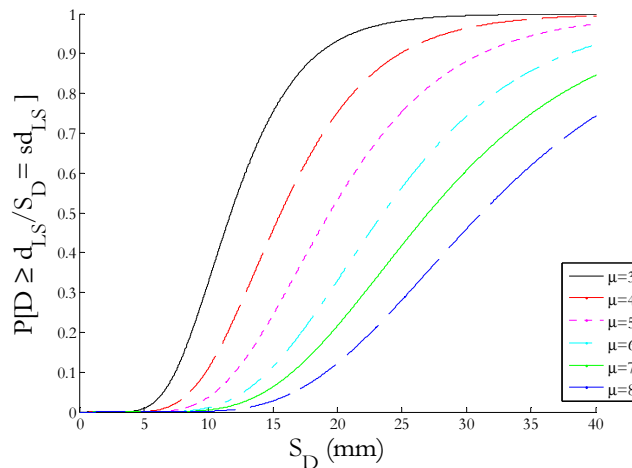
The derived fragility functions for building damage limit states are used, at a given displacement demand, to estimate the number of buildings in each damage band i.e. percent of buildings being cracked, heavily damaged and completely collapsed. The economic loss model of Bal *et al.* [2007] is used to compute the fragility functions for the economic losses of the considered buildings as a function of displacement demand. The buildings in the pre-crack states are not considered for the economic loss computation. Fig. 4.1. shows the fragility functions for damage limit states of two storey masonry buildings and economic losses of all the considered buildings. The fragility functions for the economic losses are normalised by the number of buildings ( $N$ ), for a given class, and net unit cost of single building ( $U_C$ ) for the complete replacement of the building.



**Figure 4.1.** Fragility functions for building damage limit states (left, two storey building) and economic losses (right).

#### 4.2. Code Designed Buildings

The fragility functions for the buildings designed to different levels of PGA will not differ from the functions shown in Fig. 4.1. if the drift limits for the damage states are the same because the fragility functions show the vulnerability when a given building class is subjected to a given level of seismic demand. Thus, if the displacement demand is the same on different classes of buildings designed to different levels of PGA the percentage of buildings in a given damage state will be the same for all. However, for a given earthquake different building classes will be subjected to different displacement demand and thus the percentage of buildings in a given damage state will be different for all using the same fragility functions. The fragility functions are dependent on the drift limits rather than strength thus different levels of ultimate ductility are considered for the derivation of fragility functions. Ductility levels for masonry buildings can be achieved through the use of different design schemes of reinforcements and/or confining elements [Tomazevic, 1999]. These fragility functions can be used for different retrofit schemes and design procedures as the selection of a given ductility level for a given level of design PGA will produce different consequences in terms of economic losses. Thus an optimum selection of ductility level and design PGA can be made in a way to meet a given performance criterion i.e. acceptable level of regional losses. Six levels of median ductility of 3 to 8, exceeded by 50% of the building stock on a regional scale, are considered for the derivation of fragility functions. Fig. 4.2. shows an example of fragility functions for two storey masonry buildings for increasing ultimate ductility level.



**Figure 4.2.** Fragility functions for collapse limit state of two storey masonry buildings.

## 5. CONCLUSIONS

A nonlinear static mechanics based method is proposed for the derivation of displacement-based fragility functions for buildings on regional scale capable of taking into account the uncertainty in the geometrical and material properties besides the variability in seismic demand in an explicit manner. The variability in the mechanical parameters introduced by record-to-record variability, obtained using nonlinear dynamic analysis, is also considered. The methodology is applied to the existing urban unreinforced masonry building stock of Pakistan and masonry buildings designed to different levels of PGA and ductility. Thirty case study buildings are considered with different levels of design PGA, material strength and building heights and analysed using nonlinear dynamic time history analysis using ten real accelerograms, with a total of 300 dynamic analyses, in order to develop simplified empirical expression for yield vibration of buildings. Additional six case study building classes are considered with different level of ultimate ductility. Fragility functions are derived and the parameters for the lognormal distribution functions are obtained through fitting. The derived fragility functions can be used for earthquake loss estimation within the context of seismic assessment of existing buildings and calibration for retrofit schemes and evaluation of design procedures to achieve a given target performance level on a regional scale. The procedure is coded in a numerical computing environment which can be used with high computational ease for future applications in intensity-based, scenario-based and time-based (derivation of loss exceedance curves) performance assessment. The derived fragility functions are highly dependent on the accurate definition of different inputs and their likely distribution on a regional scale. The present study assumed lognormal distribution for the application due to the absence of detailed regional data. The future application of the methodology will consider the out-of-plane vulnerability of building aggregates in the cities and out-of-plane vulnerability of individual buildings in rural areas. Application of the proposed methodology to reinforced concrete buildings is also a possible future research work.

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